EVIDENCE FOR THE OSCILLATING TWO STREAM INSTABILITY AND SPATIAL COLLAPSE OF LANGMUIR WAVES IN A SOLAR TYPE III RADIO BURST

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ABSTRACT

We present observational evidence for the oscillating two stream instability (OTSI) and spatial collapse of Langmuir waves in the source region of a solar type III radio burst. High time resolution observations from the *STEREO A* spacecraft show that Langmuir waves excited by the electron beam occur as isolated field structures with short durations \sim 3.2 ms and with high intensities exceeding the strong turbulence thresholds. These short duration events are identified as the envelope solitons which have collapsed to spatial scales of a few hundred Debye lengths. The spectra of these wave packets contain an intense peak and two sidebands, corresponding to beam-resonant Langmuir waves, and down-shifted and up-shifted daughter Langmuir waves, respectively, and low-frequency enhancements below a few hundred Hz. The frequencies and wave numbers of these spectral components satisfy the resonance conditions of the OTSI. The observed high intensities, short scale lengths, sideband spectral structures, and low-frequency enhancements strongly suggest that the OTSI and spatial collapse of Langmuir waves probably control the nonlinear beam–plasma interactions in type III radio bursts.

Key words: radiation mechanisms: general - solar wind - Sun: radio radiation

1. INTRODUCTION

Solar type III radio bursts are characterized by fast negative frequency drifts from hundreds of MHz in the solar corona to tens of kHz in the interplanetary space. Ginzburg & Zheleznyakov (1958) were the first to propose a two-step hypothesis: (1) the flare accelerated electron beam, while propagating radially outward in the solar atmosphere, excites Langmuir waves in a very narrow band around the local electron plasma frequency, $f_{\rm pe} = 9n_e^{1/2}$, by a mechanism known as the bump-on-tail instability (Bohm & Gross 1949), where n_e is the electron density in m^{-3} , and (2) subsequently, these Langmuir waves are converted into electromagnetic waves at f_{pe} as well as at $2 f_{pe}$ through some nonlinear plasma processes. Although in situ observations of electron beams (Lin 1970; Lin et al. 1973, 1981, 1986) and associated Langmuir waves (Gurnett & Anderson 1976, 1977; Kellogg et al. 1992; Gurnett et al. 1993; Thejappa et al. 1993; Hospodarsky & Gurnett 1995) in type III burst source regions confirm this hypothesis, several questions remain unanswered.

For example, Sturrock (1964) was the first to pose the following dilemma: how does the electron beam preserve the bump-on-tail distribution over distances of 1 AU and beyond against the quasi-linear relaxation, which is known to disrupt the beam within 100 km or less. The solution to this dilemma requires the effective disruption of the resonance between the Langmuir waves and the beam. Induced scattering of Langmuir waves by thermal ions (Kaplan & Tsytovich 1968), which is nothing but the electrostatic decay in the random phase approximation when $T_e > T_i$ (Bardwell & Goldman 1976; T_e and T_i are the electron and ion temperatures, respectively), has been invoked for this purpose. Stochastic growth model is also proposed for beam stabilization (Robinson & Cairns 1993). Some signatures of the parametric decay of the beamexcited Langmuir wave into a daughter Langmuir wave and an ion sound wave, which can remove the Langmuir waves out of resonance with the beam, were also observed in type III burst source regions (Lin et al. 1986; Gurnett et al. 1993;

Hospodarsky & Gurnett 1995; Thejappa & MacDowall 1998; Thejappa et al. 2003; Henri et al. 2009). However, the type III associated Langmuir waves were estimated to be very intense, and therefore, the strong turbulence processes, which can pump the Langmuir waves toward higher wave numbers k_L , were proposed as the most effective beam stabilization mechanisms. These include the oscillating two stream instability (OTSI; Papadopoulos et al. 1974; Smith et al. 1979; Goldstein et al. 1979) and related spatial collapse (Zakharov 1972; Nicholson et al. 1978; Goldman 1983).

The OTSI excites a low-frequency ion density perturbation of frequency and wave number (Ω, q) , which can be either a freely propagating ion sound wave or a strongly damped quasi-mode. This can beat with two of the initial pump waves (beam-excited Langmuir waves) of frequency and wave number (f_{pe}, k_L) and produce high-frequency down-shifted $(f_{pe} - \Omega, k_L - q)$ and up-shifted $(f_{pe} + \Omega, k_L + q)$ sidebands. The spatial collapse, on the other hand, occurs due to intensification of the localized Langmuir wave packet in the self-generated shrinking density cavity. The OTSI and spatial collapse are the focus of numerous theoretical studies (Zakharov 1972; Goldman 1984; Robinson 1997), computer simulations (Doolen et al. 1985; Russell et al. 1988), experimental investigations (Cheung & Wong 1985), ionospheric modification experiments (DuBois et al. 1993), planetary bow shock studies (Gurnett et al. 1981; Kellogg et al. 1999), and astrophysical applications (Pelletier et al. 1988).

Gurnett et al. (1981) were the first to report the possible evidence for OTSI and spatial collapse of Langmuir waves in the foreshock region of the Jovian bow shock. In the *Ulysses* data, some evidence for the Langmuir collapse (Kellogg et al. 1992), envelope solitons (Thejappa et al. 1999), and ion sound waves radiated by the burnt-out cavitons (Thejappa & MacDowall 2004) in type III burst sources have been reported. However, in *Ulysses* data, there was some uncertainty regarding millisecond spikes. In this study, we report new observations from the improved Time Domain Sampler (TDS) of the *STEREO*/WAVES experiment (Bougeret et al. 2008; improved over that of all similar high time resolution receivers

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Figure 1. Dynamic spectrum of a local type III radio burst (fast drifting emission from \sim 5 MHz down to \sim 30 kHz) and associated Langmuir waves (non-drifting emissions in the frequency interval 27–32 kHz.)

flown in earlier spacecraft in precision, linearity, sample length, and rate (Kellogg et al. 2009)). These observations provide unambiguous evidence for the sidebands and the low-frequency ion sound modes generated by the OTSI as well as for the spatial collapse of Langmuir envelope solitons in the source region of a local solar type III radio burst.

2. OBSERVATIONS

In Figure 1, the fast drifting emission feature lasting from \sim 6:00 to \sim 9:00 UT is the type III burst. This burst drifts from very high frequencies to local $f_{\rm pe} \sim 30$ kHz, indicating that it is a local event. Since its emission frequency depends on the frequency of Langmuir waves, $f_{\rm pe} \sim 9 n_e^{1/2}$, excited by the electron beam moving radially outward (n_e is the radially decreasing electron density), it occurs at lower frequencies at later times. The non-drifting emissions in the frequency interval 27–32 kHz are the Langmuir waves, excited probably by the electron beam responsible for the type III radio burst. Langmuir waves often saturate the receiver, in which case the background of the Low Frequency Receiver at the frequencies 10-2 kHz is set to a very low value. That is why, during intense Langmuir wave activity, there are no observable signals in this frequency range as seen in Figure 1. It is known that the Langmuir waves associated with type III bursts are usually very bursty (Gurnett & Anderson 1976, 1977; Gurnett et al. 1993). In Figure 2, we present the frequency-time spectrogram in a narrow frequency range, which clearly shows that the Langmuir waves occur as intense brief bursts with a substantial frequency spreading \sim 5 kHz with center frequency of \sim 30 kHz. This broad spectral width of Langmuir waves is indicative of nonlinear frequency broadening.

The TDS, which samples the A/C electric field component waveforms from three orthogonal antennas, has resolved these Langmuir waves into intense wave packets. Each of these wave packets contains 16,384 samples with an acquisition rate of 250,000 samples per second (a time step of 4 μ s for a total duration of 65 ms). In Figure 3(a), we present the most



Figure 2. Frequency-time spectrogram during the period of Langmuir wave activity. The Langmuir wave emissions have a very clumpy structure and substantial frequency spreading. The arrow shows the location of the current TDS event.



Figure 3. (a) The Langmuir wave packet observed by the Time Domain Sampler (TDS) during the type III event of Figure 1, (b) the narrow spectrum around the main Langmuir peak: *L*, *D*, and *U* refer to the beam-excited Langmuir wave at $f \sim f_{pe} \sim 30$ kHz, down-shifted ~29.54 kHz, and up-shifted ~30.41 kHz sidebands, respectively, and (c) low-frequency spectrum: the enhancement <450 Hz corresponds to ion sound waves.

intense wave packet captured by the E_x antenna. Since, the E_y , E_z , and E_{x-y} signals are weaker and show the same general features as the E_x signal, we analyze only the E_x signal. The peak electric field strength E_L of this event is 56.5 m Vm⁻¹ and its duration τ at the height of 1/e of the field intensity

maximum is ~3.2 ms. The narrow spectrum, presented in Figure 3(b), shows the main peak at $f_{\rm pe} \sim 30$ kHz corresponding to $n_e \sim 1.1 \times 10^7$ m⁻³. During this event, the *STEREO*/PLASTIC experiment (Galvin et al. 2008) has measured the solar wind speed $v_{\rm sw}$ as ~450 km s⁻¹ and we assume that the electron temperature T_e is ~10⁵ K. Assuming that the type III electrons propagate along the Parker's spiral field lines, we fit a frequency drift curve to the dynamic spectrum and estimate the beam speed v_b as ~0.22*c* for the Radio Astronomy Explorer density model (Fainberg & Stone 1971), where *c* is the velocity of light. These quantities yield: (1) the wave number of the beam-excited Langmuir waves $k_L = \omega_{\rm pe}/v_b \sim 2.9 \times 10^{-3}$ m⁻¹, (2) Debye length, $\lambda_{\rm De} = 69T_e^{1/2}n_e^{-1/2} \sim 6.6$ m, and (3) $k_L\lambda_{\rm De} \sim 1.9 \times 10^{-2}$.

3. DISCUSSION AND CONCLUSIONS

The normalized peak energy density W_L/n_eT_e = $\epsilon_0 E_L^2/2n_e T_e$ controls the nonlinear beam-plasma interactions. If $W_L/n_e T_e \ge (k_L \lambda_{\rm De})^2$, then the dominant nonlinear beam-plasma interactions are the OTSI (Papadopoulos et al. 1974) and the fully developed soliton formation and collapse (Nicholson et al. 1978). In the present case, this condition is easily satisfied, since the observed W_L/n_eT_e is 10^{-3} for $E_L = 56.5 \text{ m Vm}^{-1}, n_e = 1.1 \times 10^7 \text{ m}^{-3}$, and $T_e = 10^5 \text{ K}$, whereas, $(k_L \lambda_{\text{De}})^2$ is 3.5×10^{-4} . This suggests that the OTSI and spatial collapse can occur. Assuming that the wave packet is convected in the solar wind, we convert the measured timescale $\tau \sim 3.2$ ms into the spatial scale $S \sim 219\lambda_{\rm De}$ using the relation $S \sim \tau v_{sw}$ for $v_{sw} = 450$ km s⁻¹ and $\lambda_{De} \sim 6.6$ m. Here, we note that $k_L \sim 2.9 \times 10^{-3}$ m⁻¹ is less than $(m_e/m_i)^{1/2}k_{De} \sim 3.6 \times 10^{-3}$ $(m_{e,i}$ are the electron and ion masses, respectively, and $k_{\rm De} = 1/\lambda_{\rm De}$). For the beam speed, we have assumed a smooth spiral, whereas the pitch angle scattering is known to increase the path length of electrons by a factor of $\alpha = 1.3-1.7$ (Alvarez et al. 1975; Lin et al. 1973). This implies that the corrected beam speeds will be much more favorable for the OTSI and spatial collapse. The observed values of W_L/n_eT_e and $k_L\lambda_{\rm De}$ indicate that the OTSI in the present case refers to the supersonic modulational instability as studied by Zakharov et al. (1985). The half-width of the resonant Langmuir wave spectrum Δk_L can be inferred indirectly from $\Delta k_L/k_L = (\Delta v_b/v_b)(\ln 2/2N)$ (Lin et al. 1986; Benz 2002), where Δv_b is the range of initially unstable phase velocities, and $N \simeq \ln p$ is the number of linear growth times before the onset of OTSI. In the present case, the ratio of the peak electric field amplitude to the thermal background $p \sim 10^4$ yields $N \ge 9$. Thus, for $N \sim 9$ and $\Delta v_b \simeq 0.1 v_b$, we obtain $\Delta k_L/k_L \simeq 3.8 \times 10^{-3}$, and $\Delta k_L \simeq 10^{-5}$ m⁻¹ for $k_L = 2.9 \times 10^{-3}$ m⁻¹, and, subsequently, the bandwidth $\Delta \omega / \omega_{pe} = 3(k_L \lambda_{De})^2 \Delta k_L / k_L$ as ~ 3.3 × 10⁻⁶. On the other hand, the growth rate of OTSI $\Gamma/\omega_{\rm pe} \sim ((m_e/3m_i)(W_L/4n_eT_e))^{1/2}$ is ~4.3 × 10⁻⁴ for $W_L/n_eT_e \simeq 10^{-3}$, indicating that it is much higher than the bandwidth. This suggests that the pump waves are monochromatic enough for excitation of OTSI. It is important to note that for the bandwidth of initial pump waves, one should not use the spectral width from Figure 3(b), since it is severely affected by the nonlinear effects.

The peak intensity also decides whether this wave packet is a collapsing envelope soliton of the type described by Zakharov (1972) and Nicholson et al. (1978) or not. For the wave packet to be the collapsed soliton, it should satisfy the following condition

(Thornhill & ter Haar 1978; Gurnett et al. 1981):

$$\frac{W_L}{n_e T_e} \ge (\Delta k \lambda_{\rm De})^2, \tag{1}$$

where $\Delta k = 2\pi/S$ is the wavenumber characteristic of the envelope. In the present case, the observed $W_L/n_eT_e \sim 10^{-3}$ is greater than $(\Delta k \lambda_{\rm De})^2 \sim 8 \times 10^{-4}$ obtained for the spatial scale $S \sim 219\lambda_{\rm De}$. This suggests that the observed wave packet is probably the Langmuir envelope soliton, collapsed to the spatial scale of $\sim 219\lambda_{\rm De}$.

The narrow spectrum in Figure 3(b) shows, in addition to the intense peak at $\sim f_{pe}$, two sidebands, one at ~ 29.54 kHz (which is slightly less than f_{pe}) and a second one at ~ 30.41 kHz (which is slightly higher than f_{pe}). These spectral peaks are denoted as L, D, and U, respectively. The low-frequency spectrum presented in Figure 3(c) clearly shows the ion sound wave associated enhancement below 450 Hz. These observations of a strong Langmuir wave peak with upper and lower sidebands, together with low-frequency waves, are strongly suggestive of a nonlinear parametric interaction, in which, the beam-driven Langmuir wave is the pump wave, the modes corresponding to sidebands and low-frequency and wave number matching conditions are

$$f_D + f_U = 2f_L \tag{2}$$

$$k_D + k_U = 2k_L. \tag{3}$$

The frequency matching condition is easily satisfied, since the frequency shifts of the down-shifted and up-shifted modes are very symmetric with respect to the Langmuir wave pump, being ~442.5 Hz and ~427 Hz, respectively. Moreover, these frequency shifts are in good agreement with the observed frequencies of ion sound waves of <450 Hz. As far as the second matching condition is concerned, we have to estimate the wave numbers k_D and k_U using the expression for the frequency shift (Gurnett et al. 1981):

$$\Delta f = \frac{v_{\rm sw}}{2\pi\lambda_{\rm De}} (k\lambda_{\rm De})\cos\theta + f_{\rm pe}(-1 + (1 + 3(k\lambda_{\rm De})^2)^{1/2}), \quad (4)$$

where θ is the angle between \vec{k} and $\vec{v_{sw}}$, i.e., $\theta = 0$ and $\theta = \pi$ correspond to the up-shifted and down-shifted modes propagating away from and toward the Sun, respectively. Plugging the measured values of ~442.5 Hz and 427 Hz for Δf , we estimate $k_U \lambda_{\text{De}} \sim 0.03$ and $k_D \lambda_{\text{De}} \sim -0.05$, for the up-shifted and down-shifted modes, respectively. This indicates that the long wavelength pump Langmuir waves with $k_L \lambda_{\rm De} \sim 1.9 \times 10^{-2}$ are very efficiently converted into forward and backward propagating short wavelength Langmuir waves. Since the phase velocities of ion sound waves are usually less than v_{sw} , the upper limit of their wave numbers can be estimated as $q \simeq 6.3 \times 10^{-3}$ m⁻¹ and $q\lambda_{\rm De} \simeq 0.04$ for $\Omega = 450$ Hz and $v_{\rm sw} = 450$ km s⁻¹ using the relation $q = 2\pi\Omega/v_{sw}$. Thus, the matching condition $k_{D,U} = k_L \pm \vec{q}$ is easily satisfied, i.e., $|k_{D,U}| \simeq |\vec{q}|$, since $k_L \ll q$. The wave number of the ion sound waves excited by OTSI can also be estimated as $q\lambda_{\rm De} \sim (W_L/6n_eT_e)^{1/2} \sim 0.032$ for $W_L/n_eT_e \sim 10^{-3}$. This value, which is very close to the wave number ~ 0.04 , estimated using the observed frequency of the low-frequency enhancement further confirms that the lowfrequency waves are the daughter products of OTSI.

We have also computed the tricoherence (Kravtchenko-Berejnoi et al. 1995), which measures the degree of coherent four-wave coupling among these spectral components, and found it to be very high. We will publish the detailed trispectral analysis in a separate paper. This indicates that the observed spectral structure probably is not due to arbitrary eigenmodes as argued by Ergun et al. (2008), but due to the wave modes involved in four-wave OTSI. This further implies that the spatial collapse in the present case probably follows the OTSI as studied by Zakharov (1972).

Thus, we conclude that (1) the TDS of the STEREO/WAVES experiment has captured a very coherent and intense Langmuir wave packet in the source region of a local type III radio burst, (2) the normalized peak intensity of this wave packet is well above the threshold for OTSI, as well as soliton formation and spatial collapse, (3) the spectrum of this wave packet contains the characteristic signatures of the OTSI, namely, (a) a resonant peak at the local electron plasma frequency, f_{pe} , (b) downshifted sideband at a frequency slightly lower than f_{pe} , (c) upshifted sideband at a frequency slightly higher than f_{pe} , and (d) low-frequency enhancement corresponding to ion sound fluctuations; the frequencies and wave numbers of these spectral components satisfy the resonance conditions of OTSI, (4) the observed $W_L/n_e T_e \sim 10^{-3}$ and the short timescale ~3.2 ms well satisfy the criterion for the observed wave packet to be the collapsing Langmuir envelope soliton, and (5) in the present case, the spatial collapse appears to take the route of OTSI as studied by Zakharov (1972).

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REFERENCES

- Alvarez, H., Lin, R. P., & Bame, S. J. 1975, Sol. Phys., 44, 485
- Bardwell, S., & Goldman, M. V. 1976, ApJ, 209, 912
- Benz, A. O. 2002, Plasma Astrophysics, Kinetic Processes in Solar and Stellar Coronae (Astrophysics and Space Science Library, Vol. 279; Dordrecht: Kluwer)
- Bohm, D., & Gross, E. P. 1949, Phys. Rev., 75, 1851
- Bougeret, J.-L., Goetz, K., Kaiser, M. L., et al. 2008, Space Sci. Rev., 136, 487

- Cheung, A. Y., & Wong, A. Y. 1985, Phys. Rev. Lett., 55, 1880
- Doolen, G. D., DuBois, D. F., & Rose, H. A. 1985, Phys. Rev. Lett., 54, 804
- DuBois, D. F., Hanssen, A., Rose, H. A., & Russell, D. 1993, J. Geophys. Res., 98, 17543
- Ergun, R. E., Malaspina, D. M., Cairns, I. H., et al. 2008, Phys. Rev. Lett., 101, 051101
- Fainberg, J., & Stone, R. G. 1971, Sol. Phys., 17, 392
- Galvin, A. B., Kistler, L. M., Popecki, M. A., et al. 2008, Space Sci. Rev., 136, 437
- Ginzburg, V. L., & Zheleznyakov, V. V. 1958, SvA, 2, 653
- Goldman, M. V. 1983, Sol. Phys., 89, 403
- Goldman, M. V. 1984, Rev. Mod. Phys., 66, 709 Goldstein, M. L., Smith, R. A., & Papadopoulos, K. 1979, ApJ, 237, 683
- Gurnett, D. A., & Anderson, R. R. 1976, Science, 194, 1159
- Gurnett, D. A., & Anderson, R. R. 1970, Science, 194, 1139 Gurnett, D. A., & Anderson, R. R. 1977, J. Geophys. Res., 82, 632
- Gurnett, D. A., Wanderson, K. K. 1977, J. Goophys. Res., 62, 652 Gurnett, D. A., Hospodarsky, G. B., Kurth, W. S., Williams, D. J., & Bolton,
- S. J. 1993, J. Geophys. Res., 98, 5631 Gurnett, D. A., Maggs, J. E., Gallagher, D. L., et al. 1981, J. Geophys. Res., 86, 8833
- Henri, P., Briand, C., Mangeney, A., et al. 2009, J. Geophys. Res., 114, A03103
- Hospodarsky, G. B., & Gurnett, D. A. 1995, Geophys. Res. Lett., 22, 1161
- Kaplan, S. A., & Tsytovich, V. N. 1968, SvA, 11, 956
- Kellogg, P. J., Goetz, K., Howard, R. L., & Monson, S. 1992, Geophys. Res. Lett., 19, 1303
- Kellogg, P. J., Goetz, K., Monson, S. J., & Bale, S. D. 1999, J. Geophys. Res., 104, 17069
- Kellogg, P. J., Goetz, K., Monson, S. J., et al. 2009, J. Geophys. Res., 114, A01107
- Kravtchenko-Berejnoi, V., Lefeuvre, F., Krasnoselskikh, V., & Lagoutte, D. 1995, Signal Process., 42, 291
- Lin, R. P. 1970, Sol. Phys., 12, 266
- Lin, R. P., Evan, L. G., & Fainberg, J. 1973, Astrophys. Lett., 14, 191
- Lin, R. P., Levedahl, W. K., Lotko, W., Gurnett, D. A., & Scarf, F. L. 1986, ApJ, 308, 954
- Lin, R. P., Potter, D. W., Gurnett, D. A., & Scarf, F. L. 1981, ApJ, 251, 364
- Nicholson, D. R., Goldman, M. V., Hoyang, P., & Weatherall, J. C. 1978, ApJ, 223, 605
- Papadopoulos, K., Goldstein, M. L., & Smith, R. A. 1974, ApJ, 190, 175
- Pelletier, G., Sol, H., & Asseo, E. 1988, Phys. Rev. A, 38, 2552
- Robinson, P. A. 1997, Rev. Mod. Phys., 69, 507
- Robinson, P. A., & Cairns, I. H. 1993, ApJ, 407, 790
- Russell, D. A., DuBois, D. F., & Rose, H. A. 1988, Phys. Rev. Lett., 60, 581
- Smith, R. A., Goldstein, M. L., & Papadopoulos, K. 1979, ApJ, 234, 348
- Sturrock, P. A. 1964, in Proc. AAS- NASA Symp. Physics of Solar Flares, ed. W. N. Hess (NASA SP-50; Washington, DC: Sci. and Tech. Info. Div.), 357
- Thejappa, G., Goldstein, M. L., MacDowall, R. J., Papadopoulos, K., & Stone, R. G. 1999, J. Geophys. Res., 104, 28279
- Thejappa, G., Lengyel-Frey, D., Stone, R. G., & Goldstein, M. L. 1993, ApJ, 416, 831
- Thejappa, G., & MacDowall, R. J. 1998, ApJ, 498, 465
- Thejappa, G., & MacDowall, R. J. 2004, Nonlinear Process. Geophys., 11, 411
- Thejappa, G., MacDowall, R. J., Scime, E. E., & Littleton, J. E. 2003, J. Geophys. Res., 108, 1139
- Thornhill, S. G., & ter Haar, D. 1978, Phys. Rep., 43, 43
- Zakharov, V. E. 1972, Sov. Phys.—JETP, 35, 908
- Zakharov, V. E., Musher, S. L., & Rubenchik, A. M. 1985, Phys. Rep., 129, 285